Chi-Square (χ^2) Test of Association

Chi-Square Test of Association or Chi-Square Test of Contingency Tables: Analysis of Association Between Two Nominal or Categorical Variables

With the goodness-of-fit test one is interested in determining whether a given distribution of data follows an expected pattern. For the test of association, however, one is interested in learning whether two (or more) categorical variables are related. Typically one will find two categorical variables depicted in a contingency table (a cross-tabulation of the frequencies for various combinations of the variables). Note that contingency tables are referred to as 2-by-3, 3-by-3, etc. where the numerals are determined by the number of rows (R) and columns (C) in the table. If, for example, there is a table with two rows and two columns, the table is a R x C or 2 x 2 (2-by-2) table.

1. Example with Tenure Status and Policy Support

At issue in the following research question is whether the policy of allowing college faculty to take-on outside consultation for a fee is supported uniformly between tenured and untenured faculty. The data are as follows (example taken from D. E. Hinkle et al., 1979, Applied statistics for he behavioral sciences, Rand McNally):

Table 1
Policy Support by Tenure Status

	Support Policy	Do not Support Policy
Tenured	88	17
Nontenured	84	11

Total = 200.

2. Hypotheses

The null hypothesis states that there is no relationship between the two variables, i.e., that support for the consulting policy is independent of the tenure status of the faculty; or, that there is no difference between tenured and nontenured faculty regarding their support of the consulting policy.

H_0: distribution$_{tenured}$ = distribution$_{nontenured}$ (or the distributions are equal)

or

H_0: variable A (policy support) is independent of variable B (tenure status)

and the alternative hypothesis is:

H_1: some difference in the distributions

or

H_1: variables A and B are associated, not independent

3. Determining Expected Values

Expected values are determined by the column and row marginal frequencies. Marginal frequencies are pointed out below.

Table 2
Row and Column Totals

	Support Policy	Do not Support Policy	Marginal Row Frequencies
Tenured	88	17	88 + 17 = **105**
Nontenured	84	11	84 + 11 = **95**
Marginal Column Frequencies	88 + 84 = **172**	17 + 11 = **28**	Grand Total = 172 + 28 = **200**

Total = 200.

The following formula can be used to calculate expected frequencies for a given row r and column c, e.g., r = 1 and c = 1, which corresponds to cell "Tenured" and "Support Policy."

$$E_{rc} = \frac{(row_r total)(column_c total)}{N}$$

where $\mathbf{E}_{rc}$ is the expected value for row r and column c, $\mathbf{row}_r$ **total** is the marginal frequency for row r, $\mathbf{column}_c$ **total** is the marginal frequency for column c, and **N** is the total sample size.

For the current example, the expected values are:

a.. r = 1, c = 1 (tenured and support policy):

$$E_{11} = \frac{(105)(172)}{200} = \frac{18060}{200} = 90.3$$

b. r = 1, c = 2 (tenured and do not support policy):

$$E_{12} = \frac{(105)(28)}{200} = \frac{2940}{200} = 14.7$$

c. r = 2, c = 1 (nontenured and support policy):

$$E_{21} = \frac{(95)(172)}{200} = \frac{16340}{200} = 81.7$$

d. r = 2, c = 2 (nontenured and do not support policy):

$$E_{22} = \frac{(95)(28)}{200} = \frac{2660}{200} = 13.3$$

Table 3
Row and Column Totals and Expected Values

	Support Policy	Do not Support Policy	Marginal Row Freq.
Tenured	88 (**90.3**)	17 (**14.7**)	105
Nontenured	84 (**81.7**)	11 (**13.3**)	95
Marginal Column Freq.	172	28	200

Note: Expected values in parentheses.

4. Calculating χ^2 (chi-square)

The chi-square test of association statistic used to test H_0 can be calculated using the following formula:

$$\chi^2 = \sum \frac{(O_{rc} - E_{rc})^2}{E_{rc}}$$

The chi-square test of association formula can be explained as follows:

(1) rc = the unique cells or categories in the table of frequencies;
(2) O = the observed frequency in cell rc;
(3) E = the expected frequency in cell rc;
(4) $\sum$ = a summation sign—add up all squared terms once division has occurred;

The expected frequencies, E_{rc}, are determined in the manner demonstrated above in part (b).

The value of χ^2 is obtained as follows:

$$\chi^2 = \frac{(88-90.3)^2}{90.3} + \frac{(84-81.7)^2}{81.7} + \frac{(17-14.7)^2}{14.7} + \frac{(11-13.3)^2}{13.3}$$

$$= \frac{5.29}{90.3} + \frac{5.29}{81.7} + \frac{5.29}{14.7} + \frac{5.29}{13.3}$$

$$= 0.06 + 0.06 + 0.36 + 0.40$$

$$= 0.88$$

The χ^2 distributions are (a) positively skewed, (b) have a minimum of zero, and (c) have just one parameter which is their degree of freedom (df).

5. Degrees of freedom

The df for association chi-squares is defined as:

df (or ν) = (R - 1)(C - 1)

where R is the number of rows present and C is the number of columns present.

Since there were two rows and two columns in the example data, there is

df = (2 - 1)(2 - 1) = 1.

6. Testing H_0

To statistically test the tenability of the null hypothesis, one must determine whether the calculated value of χ^2 exceeds what would be expected by chance given that H_0 is true, i.e., does the calculated χ^2 exceed the critical value of χ^2?

The critical χ^2 or ${}_{crit}\chi^2$, can be found in critical χ^2 table. If $\alpha = .05$, the critical value for the example data is

${}_{crit}\chi^2 = 3.84$.

To test H_0, simply compare the obtained χ^2 against the critical, and if the obtained is larger, then reject H_0.

7. Decision Rule

If $\chi^2 \geq {}_{crit}\chi^2$, then reject H_0, otherwise FTR H_0.

With the current example, the decision rule is:

If $0.88 \geq 3.84$, then reject H_0, otherwise FTR H_0.

So fail to reject the null (at alpha equal to .05) and conclude that policy support does not depend upon tenure status.

8. APA Style

For a test of association it is better to report results in table format rather than text. Below is an example of table format.

Table 4
Results of Chi-square Test and Descriptive Statistics for Dropout Status by Sex

Policy Status	Tenure Status	
	Tenured	Non-tenured
Support	88 (84%)	84 (88%)
Non-support	17 (16%)	11 (12%)

Note. $\chi^2 = 0.88$, df = 1. Numbers in parentheses indicate column percentages.
*p < .05

The test of association results indicate that one's decision to support the policy of consultations does not appear to be statistically associated with one's tenure status; the results show no statistically significant difference in support rates between tenured and non-tenured faculty. These results suggest that the rate of policy support is similar for tenured and non-tenured faculty.

9. Example with Abortion Support by Party Identification

Table 5 shows Gallup's (May 23, 2011) reported split between pro-life and pro-choice among political party identification lines.

Table 5
Abortion Support by Party Identification

Abortion Stance	Republican	Independent	Democrat
Pro-life	13	8	5
Pro-choice	5	9	13

N = 53

Do the data show evidence of a difference in support among the three political groups?

10. Hypotheses

The null hypothesis states that there is no relationship between the two variables or no difference in support choices among the three groups.

H_0: Abortion Stance is independent of Political Party, or
H_0: Abortion Stance Distribution is the same across Political Parties

H_1: Abortion Stance and Political Party are associated, or
H_1: Abortion Stance Distribution varies across Political Parties

11. Determining Expected Values

Expected values are determined by the column and row marginal frequencies. Marginal frequencies are pointed out below.

Table 6
Abortion Support by Party Identification: Row and Column Totals

Abortion Stance	Republican	Independent	Democrat	**Row Totals**
Pro-life	13	8	5	**26**
Pro-choice	5	9	13	**27**
Column Totals	**18**	**17**	**18**	

Expected values entered in parentheses below. Recall that expected values are determined by this formula:

Expected Value = (Row Total × Column Total) / Overall Total

Example for Independent, Pro-life cell: (17 × 26) / 53 = 442 / 53 = 8.34

Table 7
Abortion Support by Party Identification: Row and Column Totals and Expected Values

Abortion Stance	Republican	Independent	Democrat	**Row Totals**
Pro-life	13 **(8.83)**	8 **(8.34)**	5 **(8.83)**	**26**
Pro-choice	5 **(9.17)**	9 **(8.66)**	13 **(9.17)**	**27**
Column Totals	**18**	**17**	**18**	**N=53**

Note: Expected values in parentheses.

12. Calculating χ^2 (chi-square)

The chi-square test of association statistic used to test H_0 can be calculated using the following formula:

$$\chi^2 = \sum \frac{(O_{rc} - E_{rc})^2}{E_{rc}}$$

The chi-square test of association formula can be explained as follows:

(1) rc = the unique cells or categories in the table of frequencies;
(2) O = the observed frequency in cell rc;
(3) E = the expected frequency in cell rc;
(4) $\sum$ = a summation sign—add up all squared terms once division has occurred;

The expected frequencies, E_{rc}, are determined in the manner demonstrated above.

The value of χ^2 is obtained as follows:

Table 9
Abortion Support by Party Identification: Row and Column Totals and Expected Values

Abortion Stance	Republican	Independent	Democrat	**Row Totals**
Pro-life	13 **(8.83)**	8 **(8.34)**	5 **(8.83)**	**26**
Pro-choice	5 **(9.17)**	9 **(8.66)**	13 **(9.17)**	**27**
Column Totals	**18**	**17**	**18**	**N=53**

Note: Expected values in parentheses.

$$\chi^2 = \frac{(13-8.83)^2}{8.83} + \frac{(8-8.34)^2}{8.34} + \frac{(5-8.83)^2}{8.83} + \frac{(5-9.17)^2}{9.17} + \frac{(9-8.66)^2}{8.66} + \frac{(13-9.17)^2}{9.17}$$

$$= \frac{17.387}{8.83} + \frac{0.115}{8.34} + \frac{14.670}{8.83} + \frac{17.387}{9.17} + \frac{0.115}{8.66} + \frac{14.670}{9.17}$$

$$= 1.967 + 0.014 + 1.661 + 1.896 + 0.013 + 1.600$$

$$= 7.151$$

13. Degrees of freedom

The df for association chi-squares is defined as:

df (or ν) = (R - 1)(C - 1)

Since there were two rows and three columns in the example data, there is

df = (2 - 1)(3 - 1) = 2.

14. Testing H_0

To statistically test the tenability of the null hypothesis, one must determine whether the calculated value of χ^2 exceeds what would be expected by chance given that H_0 is true, i.e., does the calculated χ^2 exceed the critical value of χ^2?

The critical χ^2 or ${}_{crit}\chi^2$, can be found in critical χ^2 table. If $\alpha = .05$, the critical value for the example data is

${}_{crit}\chi^2 = 5.99$.

To test H_0, simply compare the obtained χ^2 against the critical, and if the obtained is larger, then reject H_0.

15. Decision Rule

If $\chi^2 \geq {}_{crit}\chi^2$, then reject H_0, otherwise FTR H_0.

With the current example, the decision rule is:

If 7.151 ≥ 5.99, then reject H_0, otherwise FTR H_0.

So reject the null (at alpha equal to .05) and conclude that abortion support varies across political parties.

16. APA Style

Table 10
Results of Chi-square Test and Descriptive Statistics for Abortion Support by Party Identification

	Party Identification		
Abortion Stance	Republican	Independent	Democrat
Pro-life	13 (72%)	8 (47%)	5 (28%)
Pro-choice	5 (28%)	9 (53%)	13 (72%)

Note. $\chi^2 = 7.15$*, df = 2. Numbers in parentheses indicate column percentages.
*p < .05

Chi-square results show a statistically significant difference in abortion positions among the three political groupings. Republicans are more likely to support a pro-life stance, independents appear to have roughly even split between pro-life and pro-choice, and Democrats are more likely to support a pro-choice stance.

17. Exercises

1. A researcher wishes to determine whether an experimental treatment (RPT) enhances achievement and academic self-efficacy. The researcher must use two intact classes for the experiment since random assignment is not possible. Good research requires that the experimental and control groups be as equivalent as possible at the start of the experiment to ensure adequate internal validity. To help establish that the two classes are equivalent, the researcher plans to collect IQ and ITBS scores to determine whether a statistical difference exists between the two groups on these measures. In addition, the researcher will try to show that the two groups also have similar racial distributions. The following data are collected for the two classes:

	Black	Hispanic	White
Class 1	15	7	10
Class 2	12	6	17

(a) What are the null and alternative hypotheses?
(b) What are the expected frequencies?
(c) What is the obtained and critical chi-square statistics and df if alpha is set at the .05 level?
(d) What is the decision rule?
(e) What are the results of this test?

2. Is there a relationship between high school program of study and whether the student eventually dropped out of college? Some educators argue that students who study under college preparatory programs are much better prepared for college than are students who studied under general education programs or vocational education programs. Listed below are dropout figures for students enrolled in a medium sized, midwestern university. Determine whether dropping out is related to program of study in high school.

High School Program of Study	Dropped Out of College	Graduated from College
Vocational	289	323
General	334	456
College Prep.	230	698

(a) What are the null and alternative hypotheses?
(b) What are the expected frequencies?
(c) What is the obtained and critical chi-square statistics and df if alpha is set at the .01 level?
(d) What is the decision rule?
(e) What are the results of this test?

3. A research psychologist wants to investigate the impact of instructor feedback upon mastery of a complex learning task. Four groups of ten subjects each are selected to participate. One group receives only positive feedback, another only negative feedback. The third receives both positive and negative feedback, the fourth receives no feedback at all. The following are the results:

Type of Feedback	Successful	Nonsuccessful
Positive	6	4
Negative	4	6
Both	8	2
None	3	7

(a) What are the null and alternative hypotheses?
(b) What are the expected frequencies?
(c) What is the obtained and critical chi-square statistics and df if alpha is set at the .01 level?
(d) What is the decision rule?
(e) What are the results of this test?

Answers are posted below.

18. Computer answers for Exercises

(1) Class by Race

(a) What are the null and alternative hypotheses?

Ho: Racial distribution is same across classes.
Ha: Racial distributions differ across classes.

(b) What are the expected frequencies?

[Stata command: tabi 15 7 10 \ 12 6 17, chi2 exp]

See numbers in bold below for expected frequencies.

row	col 1	2	3	Total
1	15	7	10	32
	12.9	**6.2**	**12.9**	32.0
2	12	6	17	35
	14.1	**6.8**	**14.1**	35.0
Total	27	13	27	67
	27.0	13.0	27.0	67.0

(c) What is the obtained and critical chi-square statistics and df if alpha is set at the .05 level?

χ^2 = 2.0949, critical χ^2 = 5.991, p-value = 0.351

(d) What is the decision rule?

If $\chi^2 \geq$ critical χ^2 reject Ho otherwise fail to reject.

Since 2.09 is less than 5.991, fail to reject.

also

If $p \leq \alpha$ reject Ho, otherwise fail to reject Ho

Since .351 is not less than .05, fail to reject Ho

(e) What are the results of this test?

There is not a statistically significant difference in racial composition between the two classes. Results of the χ^2 test of association suggest that both classes appear to have similar racial compositions.

(2) Dropout by Program of Study

(a) What are the null and alternative hypotheses?

Ho: College dropout proportions are the same across high school programs of study.
Ha: College dropout proportions differ across high school programs of study.

(b) What are the expected frequencies?

```
[Stata command:  tabi 289 323 \334 456\ 230 698, chi2 exp]
```

See numbers in bold below for expected frequencies.

```
           |          col
       row |         1          2 |     Total
-----------+----------------------+----------
         1 |       289        323 |       612
           |     224.0      388.0 |     612.0
-----------+----------------------+----------
         2 |       334        456 |       790
           |     289.2      500.8 |     790.0
-----------+----------------------+----------
         3 |       230        698 |       928
           |     339.7      588.3 |     928.0
-----------+----------------------+----------
     Total |       853      1,477 |     2,330
           |     853.0    1,477.0 |   2,330.0

          Pearson chi2(2) =  96.5579   Pr = 0.000
```

(c) What is the obtained and critical chi-square statistics and df if alpha is set at the .01 level?

χ^2 = 96.5579, critical χ^2 = 9.210, p-value < 0.000

(d) What is the decision rule?

If $\chi^2 \geq$ critical χ^2 reject Ho otherwise fail to reject.

Since 96.5579 is greater than 9.210, reject.

also

If $p \leq \alpha$ reject Ho, otherwise fail to reject Ho

Since .000 is less than .01, reject Ho

(e) What are the results of this test?

There is a statistically significant difference in dropout proportions across high school programs of study. Students in the vocation and general programs of study demonstrated graduation percentage of 52.78% to 57.72%, but those college preparatory program of study had a graduation percentage of 75.22%.

(3) Success by Type of Feedback

(a) What are the null and alternative hypotheses?

Ho: Success of complex learning task is independent of feedback type.
Ha: Success of complex learning task differs across feedback type.

(b) What are the expected frequencies?

```
[Stata command:  tabi 6 4 \4 6\ 8 2 \ 3 7, chi2 exp]
```

See numbers in bold below for expected frequencies.

```
           |          col
       row |         1          2 |     Total
-----------+----------------------+----------
         1 |         6          4 |        10
           |       5.3        4.8 |      10.0
-----------+----------------------+----------
         2 |         4          6 |        10
           |       5.3        4.8 |      10.0
-----------+----------------------+----------
         3 |         8          2 |        10
           |       5.3        4.8 |      10.0
-----------+----------------------+----------
         4 |         3          7 |        10
           |       5.3        4.8 |      10.0
-----------+----------------------+----------
     Total |        21         19 |        40
           |      21.0       19.0 |      40.0

          Pearson chi2(3) =   5.9148   Pr = 0.116
```

(c) What is the obtained and critical chi-square statistics and df if alpha is set at the .01 level?

χ^2 = 5.9148, critical χ^2 = 11.345, p-value = 0.116

(d) What is the decision rule?

If $\chi^2 \geq$ critical χ^2 reject Ho otherwise fail to reject.

Since 5.9148 is less than 11.345, fail to reject Ho.

also

If $p \leq \alpha$ reject Ho, otherwise fail to reject Ho

Since .116 is greater than .01, fail to reject Ho

(e) What are the results of this test?

There is not a statistically significant difference in complex learning task success rates across feedback conditions. Results suggest that student success at learning a complete learning task are similar no matter which learning condition they experienced.